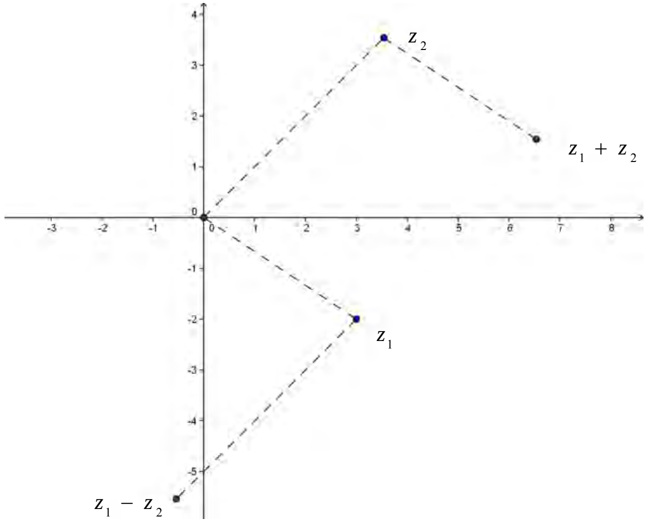
Question			Answer	Marks	Guidance	
1			$2x\left(x^2-5\right)\equiv\left(x-2\right)\left(Ax^2+Bx+C\right)+D$	M1	Evidence of comparing coefficients, or multiplying out the RHS, or substituting. May be implied by A = 2 or D = - 4	
			Comparing coefficients of x^3, $A=2$	B1		
			Comparing coefficients of x^2, $B-2A=0\Rightarrow B=4$	B1		
			Comparing coefficients of x, $C-2B=-10\Rightarrow C=-2$	B1		
			Comparing constants, $D-2C=0\Rightarrow D=-4$	B1	Unidentified, max 4 marks.	
				[5]		
2			$z=\frac{3}{2}$ is a root $\Rightarrow (2z-3)$ is a factor.	M1	Use of factor theorem, accept $2z+3, z\pm\frac{3}{2}$	
			$\Rightarrow (2z-3)\left(z^2+bz+c\right)=\left(2z^3+9z^2+2z-30\right)$	M1	Attempt to factorise cubic to linear x quadratic	
			Other roots when $z^2+6z+10=0$	M1	Compare coefficients to find quadratic (or other valid complete method leading to a quadratic)	
				A1	Correct quadratic	
			$z=\frac{-6\pm\sqrt{36-40}}{2}$	M1	Use of quadratic formula (or other valid method) in their quadratic	
			$=-3+\text{j}$ or $-3-\text{j}$	A1	oe for both complex roots FT their 3-term quadratic provided roots are complex.	
			OR $\frac{3}{2}+\beta+\gamma=-\frac{9}{2}, \frac{3}{2}\beta\gamma=15$, **or** $\frac{3}{2}\beta+\beta\gamma+\frac{3}{2}\gamma=1$	M1	Two root relations (may use α)	
			$\beta+\gamma=-6, \beta\gamma=10$	M1	leading to sum and product of unknown roots	
			$z^2+6z+10=0$,	M1	and quadratic equation	
				A1	which is correct	
			$z=\frac{-6\pm\sqrt{36-40}}{2}$	M1	Use of quadratic formula (or other valid method) in their quadratic	
			$=-3+\text{j}$ or $-3-\text{j}$	A1	oe For both complex roots FT their 3-term quadratic provided roots are complex.	
			or roots must be complex, so $a\pm bj, 2a=-6, 9+b^2=10$	M1	SCM0B1 if conjugates not justified	
			$z=-3+j, z=-3-j$	A1		
				[6]		

Question			Answer	Marks	Guidance	
3	(i)		$-2-4p=0$ $\Rightarrow\ p=-\frac{1}{2}$	M1 B1 **[2]**	Any valid row x column leading to *p*	
3	(ii)		$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{N}^{-1}\begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$	M1	Attempt to use $\mathbf{N}^{-1}$	Correct solution by means of simultaneous equations can earn full marks.
			$= \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ \frac{-7}{2} & \frac{-1}{2} & -6 \end{pmatrix}\begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$	M1	Attempt to multiply matrices (implied by 3x1 result)	M1 elimination of one unknown, M1 solution for one unknown
			$= \begin{pmatrix} 5 \\ -7 \\ 2 \end{pmatrix}$	A1 A1 **[4]**	One element correct All 3 correct. FT their *p*	A1 one correct, A1 all correct
4	(i)		$z_2 = 5\left(\cos\frac{\pi}{4} + \mathrm{j}\sin\frac{\pi}{4}\right)$ $= \frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}\mathrm{j}$	M1 A1 **[2]**	May be implied oe (exact numerical form)	

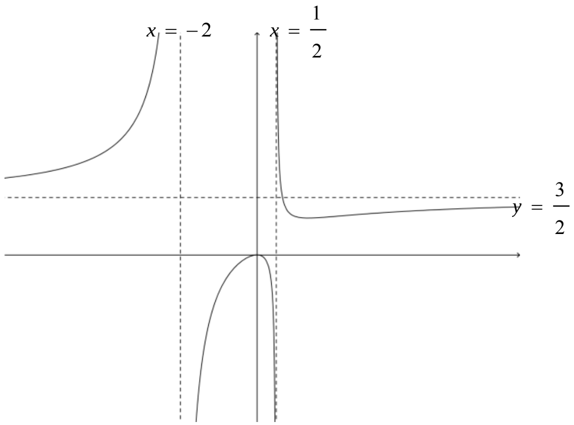
<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(ii)</td><td></td><td>$z_1 + z_2 = 3 + \frac{5\sqrt{2}}{2} + \left(-2 + \frac{5\sqrt{2}}{2}\right)\text{j} = 6.54 + 1.54\text{j}$
$z_1 - z_2 = 3 - \frac{5\sqrt{2}}{2} + \left(-2 - \frac{5\sqrt{2}}{2}\right)\text{j} = -0.54 - 5.54\text{j}$

</td><td>M1

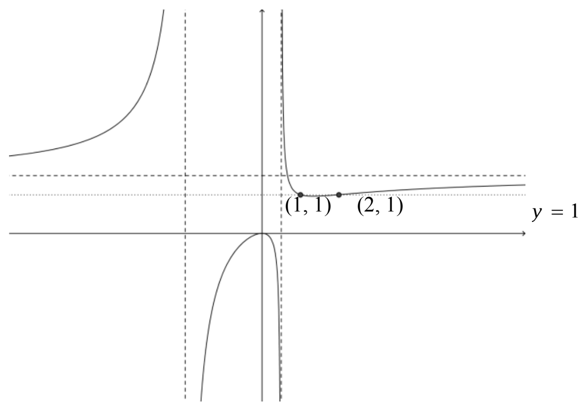
B3

[4]</td><td>Attempt to add and subtract z_1 and their z_2 - may be implied by Argand diagram

For points cao, -1 each error – dotted lines not needed.</td><td></td></tr>
<tr><td>5</td><td></td><td></td><td>$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{1}{4}\sum_{r=1}^{n}\left[\frac{1}{4r-3} - \frac{1}{4r+1}\right]$
$= \frac{1}{4}\left[\left(\frac{1}{1} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{9}\right) + \ldots + \left(\frac{1}{4n-3} - \frac{1}{4n+1}\right)\right]$
$= \frac{1}{4}\left[1 - \frac{1}{4n+1}\right]$
$= \frac{1}{4}\left[\frac{4n+1-1}{4n+1}\right] = \frac{n}{4n+1}$</td><td>M1
M1
A1
M1
A1
A1
[6]</td><td>For splitting summation into two. Allow missing 1/4
Write out terms (at least first and last terms in full)
Allow missing 1/4
Cancelling inner terms; SC insufficient working shown above,M1M0M1A1 (allow missing 1/4)
Inclusion of 1/4 justified
Honestly obtained (AG)</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance	
6			$w=\frac{x}{3}+1 \Rightarrow 3(w-1)=x$	M1		
			$x^3-5x^2+3x-6=0$			
			$\Rightarrow (3(w-1))^3-5(3(w-1))^2+3(3(w-1))-6=0$	M1	Substituting	
				A1	Correct	
			$\Rightarrow 27(w^3-3w^2+3w-1)-45(w^2-2w+1)+9w-15=0$			
			$\Rightarrow 27w^3-126w^2+180w-87=0$	A3	FT $x=3w+3, 3w\pm1$, -1 each error	
			$\Rightarrow 9w^3-42w^2+60w-29=0$	A1	cao	
			OR			
			In original equation $\sum\alpha=5, \sum\alpha\beta=3, \alpha\beta\gamma=6$	M1A1	all correct for A1	
			New roots $\mathrm{A}, \mathrm{B}, \Gamma$			
			$\sum\mathrm{A}=\frac{\sum\alpha}{3}+3, \sum\mathrm{AB}=\frac{\sum\alpha\beta}{9}+\frac{2}{3}\sum\alpha+3$			
			$\mathrm{AB}\Gamma=\frac{\alpha\beta\gamma}{27}+\frac{\sum\alpha\beta}{9}+\frac{\sum\alpha}{3}+1$	M1 A3	At least two relations attempted Correct -1 each error FT their 5,3,6	
			Fully correct equation	A1 **[7]**	Cao, accept rational coefficients here	

Question			Answer	Marks	Guidance	
7	(i)		Vertical asymptotes at $x = -2$ and $x = \frac{1}{2}$ occur when $(bx-1)(x+a) = 0$ $\Rightarrow a = 2$ and $b = 2$ Horizontal asymptote at $y = \frac{3}{2}$ so when x gets very large, $\frac{cx^2}{(2x-1)(x+2)} \to \frac{3}{2} \Rightarrow c = 3$	M1 A1 A1 A1 **[4]**	Some evidence of valid reasoning – may be implied	
7	(ii)		Valid reasoning seen Large positive x, $y \to \frac{3}{2}$ from below Large negative x, $y \to \frac{3}{2}$ from above  	M1 A1 B1 B1 **[4]**	Some evidence of method needed e.g. substitute in 'large' values with result Both approaches correct (correct b,c) LH branch correct RH branch correct Each one carefully drawn.	

Question			Answer	Marks	Guidance	
7	(iii)		$\frac{3x^2}{(2x-1)(x+2)} = 1 \Rightarrow 3x^2 = (2x-1)(x+2)$	M1	Or other valid method, to values of x (allow valid solution of inequality)	
			$\Rightarrow 0 = (x-2)(x-1)$		Explicit values of x	
			$\Rightarrow x = 1$ or $x = 2$	A1		
			From the graph $\frac{3x}{(2x-1)(x+2)} < 1$			
			for $-2 < x < \frac{1}{2}$ or $1 < x < 2$	B1 B1	FT their x=1,2 provided >1/2.	
				[4]		

Question			Answer	Marks	Guidance	
8	(i)		$\sum_{r=1}^{n}[r(r-1)-1] = \sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} r - n$	M1	Split into separate sums	
			$= \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$	M1	Use of at least one standard result (ignore 3rd term)	
				A1	Correct	
			$= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 6]$	M1	Attempt to factorise. If more than two errors, M0	
			$= \frac{1}{6}n[2n^2 - 8]$			
			$= \frac{1}{3}n[n^2 - 4]$	A1	Correct with factor $\frac{1}{3}n$ oe	
			$= \frac{1}{3}n(n+2)(n-2)$		Answer given	
				[5]		
8	(ii)		When $n = 1$,			
			$\sum_{r=1}^{n}[r(r-1)-1] = (1\times 0) - 1 = -1$			
			and $\frac{1}{3}n(n+2)(n-2) = \frac{1}{3}\times 1\times 3\times -1 = -1$			
			So true for $n = 1$	B1		
			Assume true for $n = k$	E1	Or "if true for n=k, then…"	
			$\sum_{r=1}^{k}[r(r-1)-1] = \frac{1}{3}k(k+2)(k-2)$			
			$\Rightarrow \sum_{r=1}^{k+1}[r(r-1)-1] = \frac{1}{3}k(k+2)(k-2) + (k+1)k - 1$	M1*	Add (k + 1)th term to both sides	
			$= \frac{1}{3}k^3 + k^2 - \frac{4}{3}k + k - 1$			
			$= \frac{1}{3}(k^3 + 3k^2 - k - 3)$			

Question			Answer	Marks	Guidance	
			$= \frac{1}{3}(k+1)(k^2+2k-3)$	M1dep *	Attempt to factorise a cubic with 4 terms	
			$= \frac{1}{3}(k+1)(k+3)(k-1)$	A1		
			$= \frac{1}{3}(k+1)((k+1)+2)((k+1)-2)$ But this is the given result with $n = k + 1$ replacing $n = k$.		Or $= \frac{1}{3}n(n+2)(n-2)$ where $n = k+1$; or target seen	
			Therefore if the result is true for $n = k$, it is also true for $n = k+1$.	E1	Depends on A1 and first E1	
			Since it is true for $n = 1$, it is true for all positive integers, n.	E1 **[7]**	Depends on B1 and second E1	
9	**(i)**		**Q** represents a rotation 90 degrees clockwise about the origin	B1 B1 **[2]**	 Angle, direction and centre	
9	**(ii)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ $P = (-2, 2)$	M1 A1 **[2]**	 Allow both marks for P(-2, 2) www	
9	**(iii)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix}$ l is the line $y = -x$	M1 A1 **[2]**	Or use of a minimum of two points Allow both marks for $y = -x$ www	
9	**(iv)**		$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ n is the line $y = 6$	M1 B1 **[2]**	Use of a general point or two different points leading to $\begin{pmatrix} -6 \\ 6 \end{pmatrix}$ y=6; if seen alone M1B1	

Question			Answer	Marks	Guidance	
9	(v)		$\det \mathbf{M} = 0 \Rightarrow \mathbf{M}$ is singular (or 'no inverse'). The transformation is many to one.	B1 E1 **[2]**	www Accept area collapses to 0, or other equivalent statements	
9	(vi)		$\mathbf{R} = \mathbf{Q}\mathbf{M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix}$ q is the line $y = x$	M1 A1 **[2]**	Attempt to multiply in correct order Or argue by rotation of the line $y = -x$ $y = x$ SC B1 following M0	